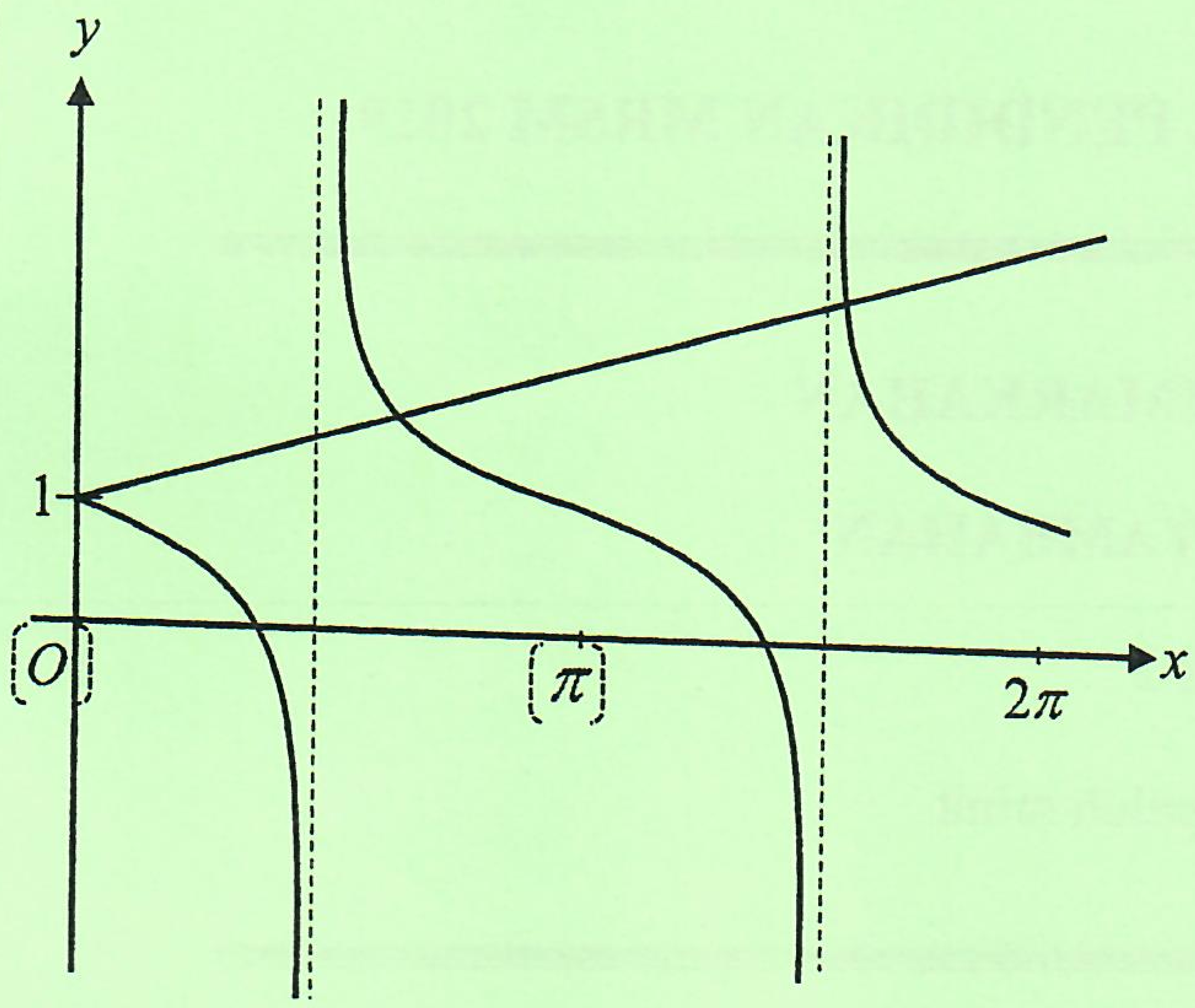
Additional Mathematics Paper 2
SPMRSM 2019

Answer Scheme

No	Solution	Scheme	Sub marks	Marks
1 (a)		P1 Shape of tangent graph P1 Shape of negative tangent graph P1 Shift upward +1. Note : 1. Do not accept sine and cosine graph. 2. Ignore graph outside the range.	3	
(b)	$y = \frac{x}{\pi} + 1$ Number of solutions = 3	N1 $y = \frac{x}{\pi} + 1$ K1 Sketch straight line $*y = \frac{x}{\pi} + 1$ with *gradient property or *y-intercept property correct. N1 3	3	6

No	Solution	Scheme	Sub marks	Marks
2	$\log_9 x^2 - \log_3(x-4) = \log_3 5$			
	$\dfrac{\log_3 x^2}{\log_3 9} - \log_3(x-4) = \log_3 5$	K1 Use change base formula $\log_a b = \dfrac{\log_c b}{\log_c a}$.		
	$\dfrac{\log_3 x^2}{2} - \log_3(x-4) = \log_3 5$			
	$\log_3 x - \log_3(x-4) = \log_3 5$	K1 Use law $\log_a m^n = n\log_a m$.		
	$\log_3\left(\dfrac{x}{x-4}\right) = \log_3 5$	K1 Use law $\log_a \dfrac{m}{n} = \log_a m - \log_a n$ or $\log_a mn = \log_a m + \log_a n$.		
	$\dfrac{x}{x-4} = 5$	K1 Equate LHS to RHS OR change logarithm form to index form.		
	$x = 5$	N1 $x = 5$	5	5

No	Solution	Scheme	Sub marks	Marks
3	$2y+20x=160$ or $12x^2+2xy=600$	P1 $2y+20x=160$ or $12x^2+2xy=600$		
	$y=80-10x$ or $x=8-\dfrac{y}{10}$ or $y=\dfrac{300}{x}-6x$	P1 seen or implied		
	$12x^2+2x^*(80-10x)=600$ or $12^*\left(8-\dfrac{y}{10}\right)^2+2^*\left(8-\dfrac{y}{10}\right)y=600$ or $2\left(\dfrac{300}{x}-6x\right)+20x=160$	K1 Eliminate x or y.		
	Factorization $(x-5)(x-15)=0$ or $(y+10)(y-30)=0$ OR **Formula** $x=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(1)(75)}}{2(1)}$ or $y=\dfrac{-(40)\pm\sqrt{(40)^2-4(1)(2100)}}{2(1)}$ OR **Completing the square** $8\left[(x-10)^2-(-10)^2-75\right]=0$ or $2\left[(y+20)^2-(20)^2-2100\right]=0$	K1 Solve quadratic equation using factorization, formula or completing the square.		
	$x=5$, $\{x=15\}$ $y=30$, $\{y=-10\}$	N1 First set value x or y.		
	Length = 30 meter Width =10 meter	N1 Length = 30 meter Width =10 meter Note: 1. OW-1 it steps to solve the quadratic equation is not shown. 2. SS-1 improper factorization is shown.	6	6

No	Solution	Scheme	Sub marks	Marks
4 (a)	(i) $\overrightarrow{SU} = 2\underline{p}$	N1 $2\underline{p}$		
	(ii) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$	K1 Use triangle law or parallelogram law to find $\overrightarrow{PR}$		
	$= 8\underline{p} + 2\underline{q}$	N1 $8\underline{p} + 2\underline{q}$	3	
(b)	$\overrightarrow{PU} = 2\underline{p} + 2\underline{q}$	K1 Use $\overrightarrow{PU} = \overrightarrow{PS} + \overrightarrow{SU}$ or $\overrightarrow{ST} = \overrightarrow{SP} + m\overrightarrow{PU}$.		
	$\overrightarrow{ST} = -2\underline{q} + m(2\underline{p} + 2\underline{q})$			
	$= 2m\underline{p} + (2m-2)\underline{q}$	N1 $2m\underline{p} + (2m-2)\underline{q}$	2	
(c)	$^{*}2m\underline{p} + (2m-2)\underline{q} = \lambda^{*}(8\underline{p} + 2\underline{q})$	K1 Use $^{*}\overrightarrow{ST} = \lambda\, ^{*}\overrightarrow{PR}$ or $^{*}\overrightarrow{PR} = \lambda\, ^{*}\overrightarrow{ST}$.		
	$^{*}2m = ^{*}8\lambda$ $\quad$ $^{*}2m - 2 = ^{*}2\lambda$	K1 Equate the coefficient of $\underline{p}$ and of $\underline{q}$ and solve.		
	$m = \frac{4}{3}$	N1 $\frac{4}{3}$	3	8

No	Solution	Scheme	Sub marks	Marks
5 (a)	$f(x)=x-75$ $\quad gf(x)=0.95x$	P1 $x-75$ or $0.95x$		
	Let $y=x-75$	K1 Find f^{-1}.		
	$x=y+75$			
	$g(y)=0.95(y+75)$			
	$g(x)=0.95(x+75)$	N1 $0.95(x+75)$	3	
(b)				
	(i) $0.95x=499.90$	K1 Equate $0.95x=499.90$.		
	$x=526.21$	N1 $x=526.21$		
	(ii) $499.90-({}^{*}526.21-75)$	K1 $499.90-({}^{*}526.21-75)$		
	48.69	N1 48.69	4	7
		Note: For correct answer only, award K1N1.		

No	Solution	Scheme	Sub marks	Marks
6 (a)	$A=\pi r^2+\pi rp$ $\pi r^2+\pi rp=160$	P1 $\pi r^2+\pi rp=160$		
	$p=\dfrac{160-\pi r^2}{\pi r}$ $V=\dfrac{1}{2}\pi r^2\left(\dfrac{160-\pi r^2}{\pi r}\right)$	K1 Subtitute $p=\dfrac{160-\pi r^2}{\pi r}$ into V.		
	$V=80r-\dfrac{\pi r^3}{2}$	N1 $V=80r-\dfrac{\pi r^3}{2}$	3	
(b)	$\dfrac{dV}{dr}=80-\dfrac{3}{2}\pi r^2$	K1 Differentiate V w.r.t r.		
	$80-\dfrac{3}{2}\pi r^2=0$	K1 Equate $\dfrac{dV}{dr}$ to 0		
	$r=\sqrt{\dfrac{160}{3\pi}}$ or equivalent	N1 $r=\sqrt{\dfrac{160}{3\pi}}$ or equivalent		
	$V=80\left(\sqrt{^*\dfrac{160}{3\pi}}\right)-\dfrac{\pi\left(\sqrt{^*\dfrac{160}{3\pi}}\right)^3}{2}$	K1 Substitute $^*r$ into V		
	219.7 // 219.75	N1 219.7 // 219.75	5	8

No	Solution	Scheme	Sub marks	Marks
7 (a)				
	(i) $^7C_6\left(\frac{4}{7}\right)^6\left(\frac{3}{7}\right)^1$	K1 Use $^7C_rp^rq^{7-r}$.		
	0.1044	N1 0.1044		
	(ii) $^7C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^7$ or $^7C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^6$	K1 Use $^7C_rp^rq^{7-r}$.		
	$1-{}^7C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^7-{}^7C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^6$ or $^7C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^5+{}^7C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^4+\ldots+{}^7C_7\left(\frac{3}{7}\right)^7\left(\frac{4}{7}\right)^0$	K1 $1-P(X=0)-P(X=1)$ or $P(X=2)+P(X=3)+\ldots+P(X=7)$		
	0.8757	N1 0.8757	5	
(b)	(i) $\frac{81-90}{12}$ or $\frac{108-90}{12}$	K1 Use of $Z=\frac{X-\mu}{\sigma}$.		
	0.2934	N1 0.2934		
	(ii) [–]1.645	P1 seen or implied		
	$\frac{t-90}{12}=-1.645$	K1 Equate $\frac{t-90}{12}=-1.645$.		
	70.26	N1 70.26	5	10

No	Solution	Scheme	Sub marks	Marks
8 (a)	(i) $-2\times m_2=-1$	K1 Use $m_1\times m_2=-1$.		
	$\dfrac{y-12}{0-4}=\dfrac{1}{2}$	K1 Use any valid method to find y-coordinate.		
	$y=10$	N1 $y=10$	3	
	(ii) $y=-\dfrac{1}{2}\left(\dfrac{x^2}{2}\right)+c$	K1 Integrate $-\dfrac{1}{2}x$ w.r.t. x.		
	$12=-\dfrac{1}{2}\left(\dfrac{4^2}{2}\right)+c$	K1 Find the value of c.		
	$y=-\dfrac{x^2}{4}+16$	N1 $y=-\dfrac{x^2}{4}+16$	3	
(b)	$\pi\int_{12}^{k}(64y-4y)\,dy=50\pi$			
	$\left[64y-\dfrac{4y^2}{2}\right]_k^{16}=50$	K1 Integrate $\pi^*(64-4y)$ and equate to 50π		
	$\left[64(16)-2(16)^2\right]-\left[64k-2k^2\right]=50$	K1 Use limit $\int_k^{16}$		
	$k^2-32k+231=0$ $(k-21)(k-11)=0$	K1 Solve quadratic equation.		
	$k=11$	N1 $k=11$	4	10

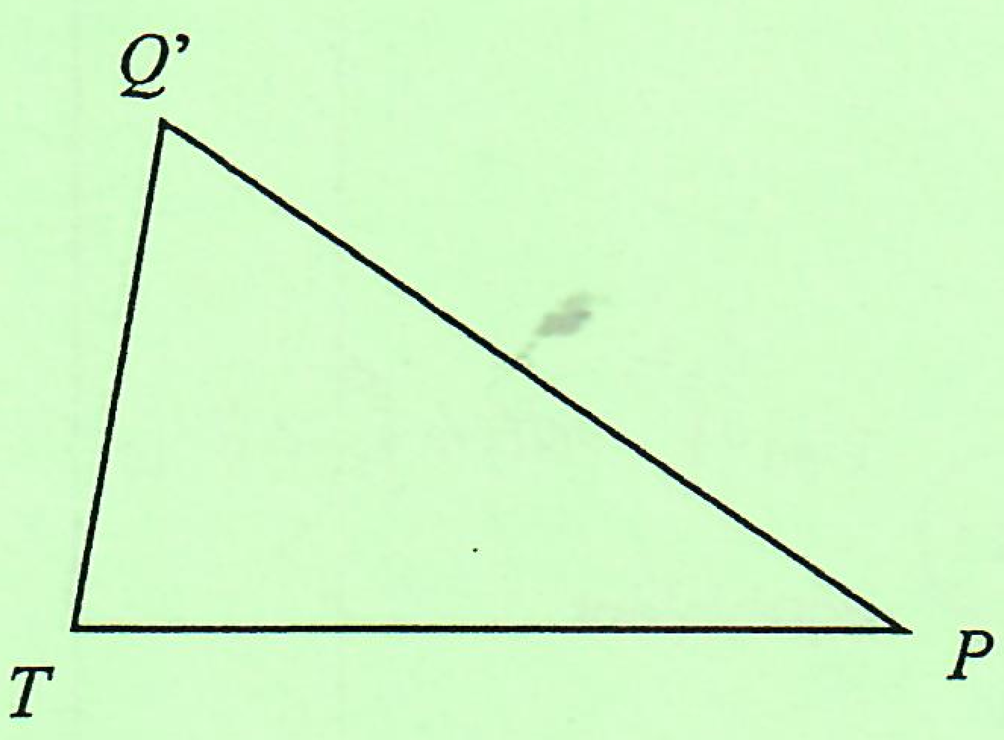
No	Solution	Scheme	Sub marks	Marks
9 (a)	(i) $-\frac{1}{3}\times m_2=-1$ $y-4=3(x-6)$ $y=3x-14$	K1 Use $m_1\times m_2=-1$. K1 Use $y-y_1=m(x-x_1)$ or any valid method. N1 $y=3x-14$		
	(ii) $\frac{2}{3}x+6=3x-14$ $\left(\frac{60}{7},\frac{82}{7}\right)$	K1 Simultaneous equation. N1 $\left(\frac{60}{7},\frac{82}{7}\right)$	5	
(b)	$\frac{0(3)+2x}{5}=9$ or $\frac{6(3)+2y}{5}=6$ $\left(\frac{45}{2},6\right)$	K1 Use ratio formulae. N1 $\left(\frac{45}{2},6\right)$ Note: For correct answer only, award K1N1.	2	
(c)	$C(12, 2)$ $\frac{1}{2}\begin{vmatrix}0 & \frac{45}{2} & 12 & 0\\ 6 & 6 & 2 & 6\end{vmatrix}$ $\frac{1}{2}\left\|(45+72)-(135+72)\right\|$ 45	N1 $C(12, 2)$ K1 Use area formula. N1 45	3	10

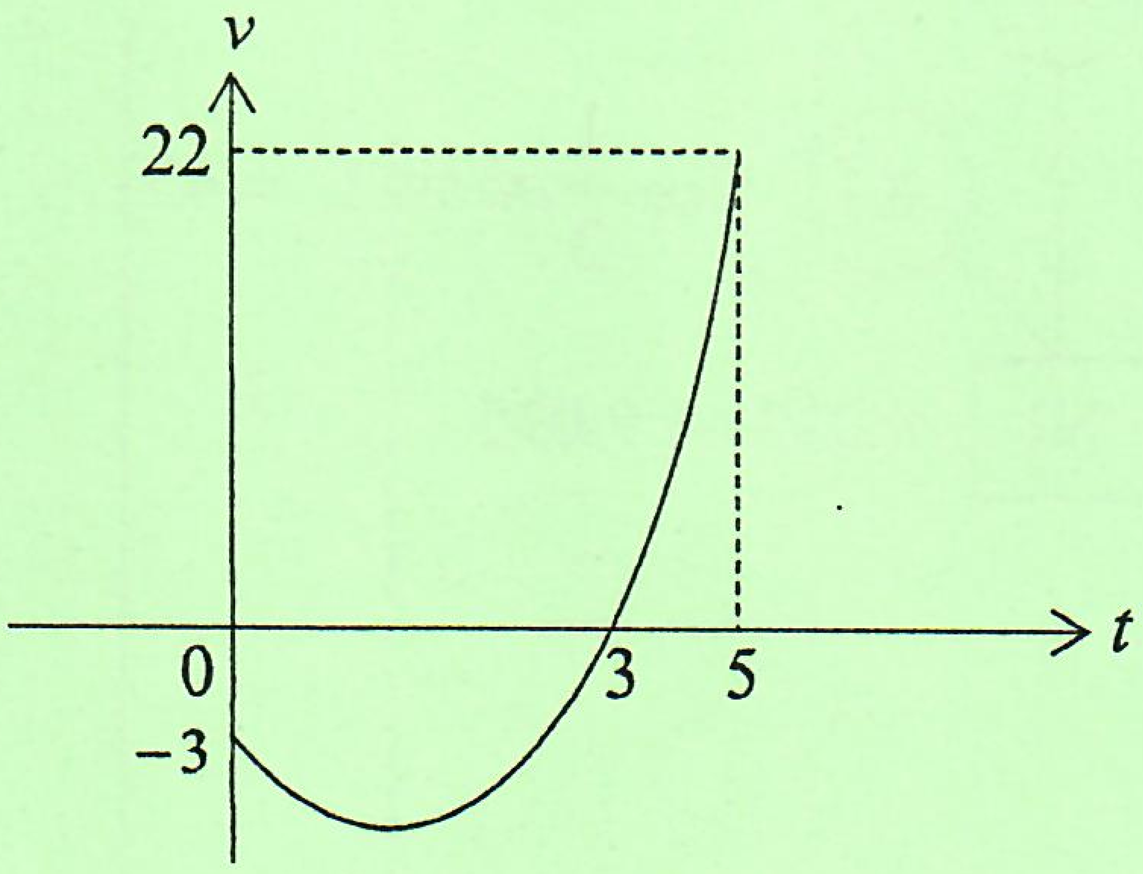
No	Solution	Scheme	Sub marks	Marks
10 (a)				

x^3	0.55	1.00	1.52	2.20	3.05	3.65
xy	87	80	73	63	51	42

No	Solution	Scheme	Sub marks	Marks
		N1 Note: at least two d.p		
		N1		
	Plot xy against x^3	K1 Plot xy against x^3 with correct axes and uniform scales.		
	*6 points plotted correctly	N1		
	Draw line of best fit	N1 Line of best fit.	5	
(b)	$xy = rx^3 + s$	P1 $xy = rx^3 + s$ seen or implied		
	(i) $r = -15.4 \leftrightarrow -13.4$	K1 Use $^*m = r$. N1 $-15.4 \leftrightarrow -13.4$		
	(ii) $s = 93.7 \leftrightarrow 95.7$	N1 $93.7 \leftrightarrow 95.7$		
	(iii) $x = 1.46 \leftrightarrow 1.56$	N1 $1.46 \leftrightarrow 1.56$	5	10
		Note : SS – 1 if, part of the scale is not uniform at the xy–axis and/or the x^3–axis from the first point to the last point *or* does not use the given scale *or* does not use graph paper.		

No	Solution	Scheme	Sub marks	Marks
11 (a)	$\angle POR = 62.67°$ or $\angle PXR = 59.52°$	P1 $\angle POR = 62.67°$ or $\angle PXR = 59.52°$ seen or implied		
	$PR^2 = 10^2 + 10^2 - 2(10)(10)\cos 62.67°$ $PR = 10.40$	K1 Use cosine rule or any valid method to find PR.		
	$Arc_{PR} = 10(1.094)$	K1 Use $s = r\theta$ to find arc PR.		
	Perimeter A = $^*10.40 + 10(1.094)$	K1 PR + arc PR.		
	21.34 // 21.35	N1 21.34 // 21.35	5	
(b)	$\cos 60.23° = \frac{5.2}{XR}$ $XR = 10.48$	K1 Use any valid method to find XR or XP.		
	$\frac{1}{2} \, ^*(10.48)^2 \, ^*(1.039)$	K1 Use any valid method to find area of sector $XPYR$.		
	$\frac{1}{2} \, ^*(10.48)^2 \, ^*\sin 59.52°$	K1 Use any valid method to find area of ΔXPR.		
	Area B $\left[\frac{1}{2} \, ^*(10.48)^2 \, ^*(1.039)\right] - \left[\frac{1}{2} \, ^*(10.48)^2 \, ^*\sin 59.52°\right]$	K1 *Area of sector $XPYR$ − *area of ΔXPR		
	$9.72 \leftrightarrow 9.78$	N1 $9.72 \leftrightarrow 9.78$	5	10

No	Solution	Scheme	Sub marks	Marks
12 (a)	(i) $\frac{6.7}{\sin 40^\circ} = \frac{9}{\sin \angle PQT}$	K1 Use sine rule.		
	$\angle PQT = 120.29^\circ$ // $120^\circ 17'$ $\angle RQT = 180^\circ - 120.29^\circ = 59.71^\circ$	N1 59.71°		
	$RT^2 = 12^2 + 6.7^2 - 2(12)(6.7)\cos 59.71^\circ$ $RT = 10.38$	K1 Use cosine rule. N1 $RT = 10.38$		
	(ii) $\frac{10.38}{\sin 59.71^\circ} = \frac{12}{\sin \angle QTR}$ $\angle QTR = 86.61^\circ$ // $86^\circ 36'$	K1 Use sine rule. N1 86.61°	6	
(b)	$\angle RTS = 180^\circ - 86.61^\circ - 19.71^\circ = 73.68^\circ$ $\frac{1}{2}(10.38)(ST)\sin^{*} 73.68^\circ = 45$ 9.035	P1 73.68° K1 Use $\frac{1}{2}ab\sin C = 45$. N1 $9.028 \leftrightarrow 9.035$	3	
(c)		N1 Triangle with $\angle PQ'T$ must be acute.	1	10

No	Solution	Scheme	Sub marks	Marks
13 (a)	$2t^2-5t-3=0$	K1 Use $v=0$ and solve.		
	$t=-\frac{1}{2}, t=3$			
	$s=\frac{2t^3}{3}-\frac{5t^2}{2}-3t+c$	K1 Integrate v w.r.t. t.		
	$^*s=\frac{2^*(3)^3}{3}-\frac{5^*(3)^2}{2}-3^*(3)$	K1 Substitute $^*t$ into $^*s$.		
	$-\frac{27}{2}//-13\frac{1}{2}//-13.5$	N1 $-\frac{27}{2}//-13\frac{1}{2}//-13.5$	4	
(b)	$4t-5<0$	K1 Use $a<0$.		
	$0\le t<\frac{5}{4}$	N1 $0\le t<\frac{5}{4}$	2	
(c)		P1 Minimum shape graph. P1 Label at least ^{*3} points.		
	$\left\|\frac{2(^*3)^3}{3}-\frac{5(^*3)^2}{2}-3(^*3)-0\right\|+$ $\left[\frac{2(5)^3}{3}-\frac{5(5)^2}{2}-3(5)-\left(\frac{2(^*3)^3}{3}-\frac{5(^*3)^2}{2}-3(^*3)\right)\right]$	K1 Use $\left\|\int_0^{^*3} v\,dt\right\|+\int_{^*3}^{5} v\,dt$ or equivalent.		
	$\frac{197}{6}//32\frac{5}{6}//32.83$	N1 $\frac{197}{6}//32\frac{5}{6}//32.83$		

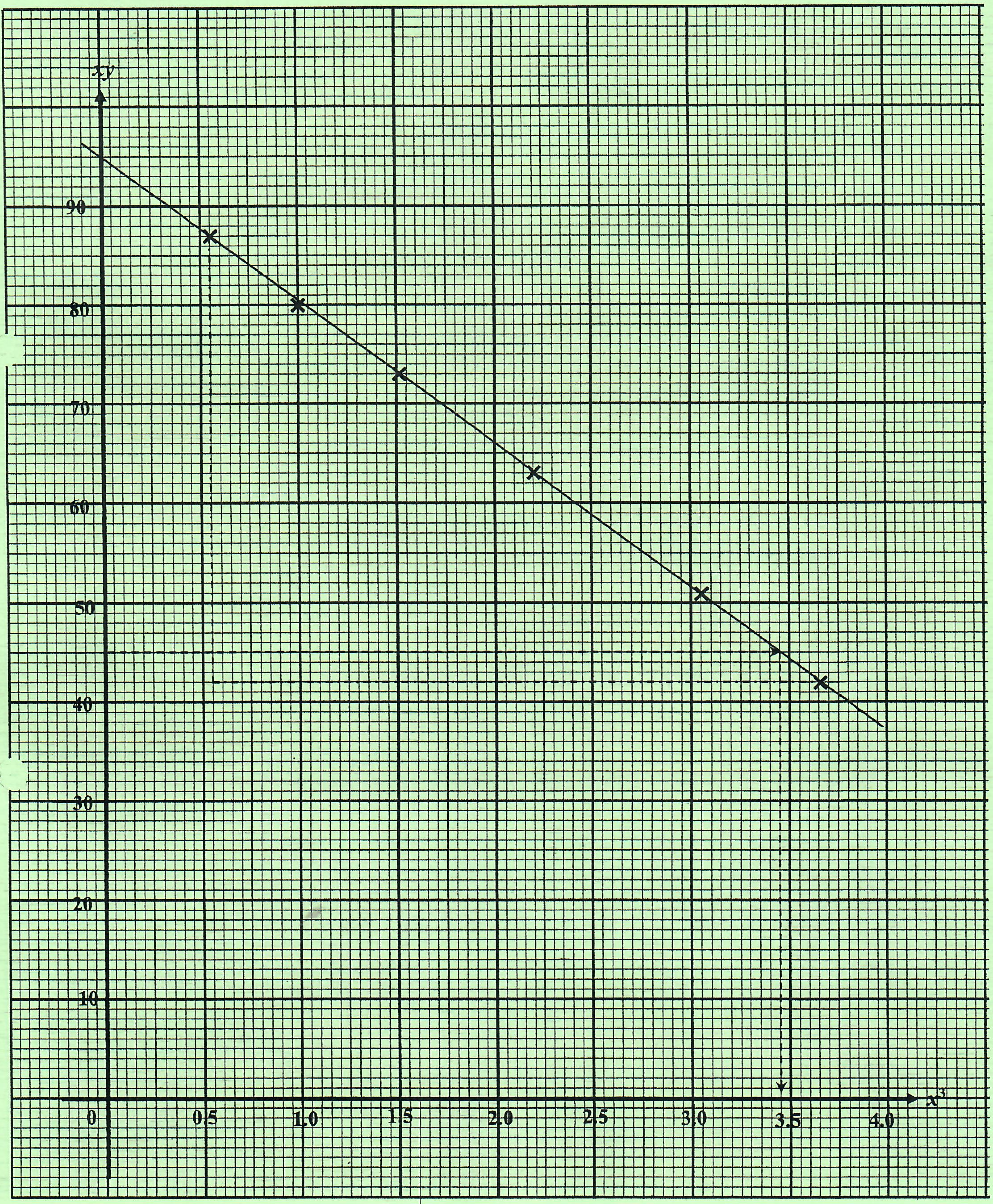
No	Solution	Scheme	Sub marks	Marks
	OR $t=0 \rightarrow s=0$ $t={}^{*}3 \rightarrow s=\frac{2({}^{*}3)^3}{3}-\frac{5({}^{*}3)^2}{2}-3({}^{*}3)=-13.5$ $t=5 \rightarrow s=\frac{2(5)^3}{3}-\frac{5(5)^2}{2}-3(5)=5\frac{5}{6}$ $\lvert s_{*3}-s_0\rvert+\lvert s_5-s_{*3}\rvert=\lvert -13.5-0\rvert+\left\lvert 5\frac{5}{6}-(-13.5)\right\rvert$ $\frac{197}{6}//32\frac{5}{6}//32.83$	K1 Use $\lvert s_{*3}-s_0\rvert+\lvert s_5-s_{*3}\rvert$ or equivalent N1 $\frac{197}{6}//32\frac{5}{6}//32.83$	4	10

No	Solution	Scheme	Sub marks	Marks
14 (a)	$y \leq 2x$	N1 $y \leq 2x$		
	$y - x \geq 100$ or equivalent	N1 $y - x \geq 100$ or equivalent		
	$x + y \leq 750$ or equivalent	N1 $x + y \leq 750$ or equivalent	3	
(b)	Refer graph.	K1 Draw correctly at least one straight line from the *inequalities involves x and y.		
		N1 Draw correctly all *straight lines.		
		N1 Region shaded correctly.	3	
(c)	(i) $300 \leq y \leq 400$	N1 $300 \leq y \leq 400$		
	(ii) Minimum point (100, 200)	N1 (100, 200)		
	$(100 + 200)(12)$	K1 Substitute any point in *shaded region into $(x + y)$.		
	RM3600	N1 3600		
		Note : SS-1 only once if in (a)(i) the symbol '=' is not used at all (ii) more than 3 inequalities given OR in (b)(i) does not use given scale (ii) axes interchanged (iii) not using graph paper.	4	10

No	Solution	Scheme	Sub marks	Marks
15 (a)	$\frac{220}{r}\times 100=110$ $r=200$ $s=\frac{187.5}{150}\times 100$ $s=125$ $\frac{t}{400}\times 100=130$ $t=520$	K1 Use $\frac{P_1}{P_0}\times 100$ N2, 1, 0 $r=200$, $s=125$, $t=520$	3	
(b)	$\frac{(110\times 100)+(^{*}125\times 80)+(105\times 60)+(130\times 40)}{280}$ 116.07	K1 Use $\bar{I}=\frac{\sum Iw}{\sum w}$ N1 116.07	2	
(c)	$\frac{U}{75000}\times 100={}^{*}116.07$ $U=87052.50$ 2808.15	K1 Use $\frac{U}{75000}\times 100={}^{*}116.07$ N1 87052.50 N1 2808.15	3	
(d)	$\frac{140}{^{*}116.07}\times 100$ 120.62	K1 Use $\frac{140}{^{*}116.07}\times 100$ N1 120.62	2	10

Graph for Question 10

x^3	0.55	1.00	1.52	2.20	3.05	3.65
xy	87	80	73	63	51	42



Graph for Question 14