

SULIT
3472/1
Matematik
Tambahan
Kertas 1
November
2022

2 jam



MAKTAB RENDAH SAINS MARA

PEPERIKSAAN AKHIR

SIJIL PENDIDIKAN MRSM 2022

PERATURAN PEMARKAHAN

MATEMATIK TAMBAHAN

Kertas 1

Dua jam

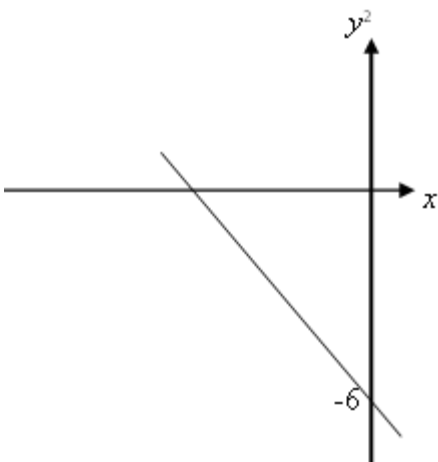
UNTUK KEGUNAAN PEMERIKSA SAHAJA

AMARAN

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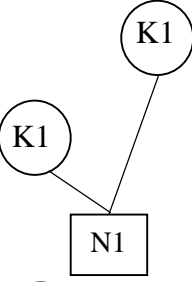
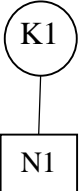
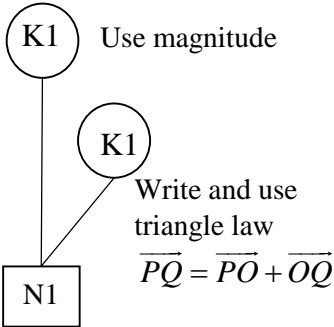
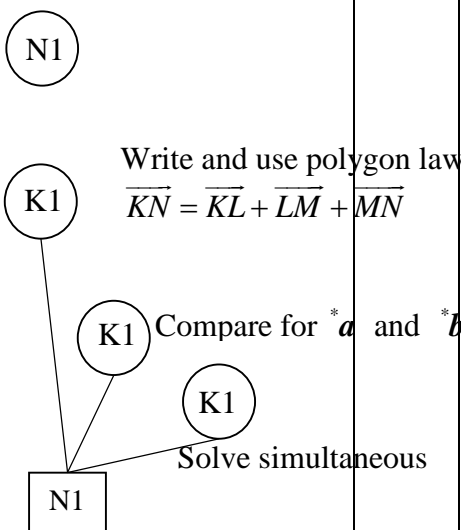
Dokumen ini mengandungi 11 halaman bercetak

ANSWER SCHEME

No	Solution	Scheme	Sub marks	Marks
1 (a)	$2^2 = \frac{p}{2}(-5) + q \text{ or } 4^2 = \frac{p}{2}(-11) + q$ $p = -4, q = -6 \text{ (both)}$	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> <div style="margin-left: 10px;">Substitute $y = 2, x = -5$ or $y = 4, x = -11$</div> </div> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> <div style="margin-left: 10px;">Solve simultaneous equations</div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 5px;">N1</div> <div>$p = -4, q = -6 \text{ (both)}$</div> </div> </div>	3	
(b)		<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 5px; margin-right: 5px;">N1</div> <div style="margin-left: 10px;">Note: Straight line with y-intercept = -6</div> </div> </div>	1	
2 (a)	$2\alpha + 2\beta = -m$ $\left\{ \begin{matrix} m = 4 \end{matrix} \right.$	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 5px;">N1</div> </div> </div>	2	
(b)	$2\alpha(2\beta) = -3 \text{ OR } \alpha = -2.3229, \beta = 0.3229 \text{ (both)}$ $x^2 + 2x - \frac{3}{4} = 0$	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> </div> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 5px;">N1</div> </div> </div>	2	
				4

<p>3</p> <p>(a)</p> <p>(b)</p> <p>(i)</p> <p>(ii)</p> <p>(iii)</p>	<p>$y = f(x)$, Vertical line test cut only one point // one to one relation // each object has only one image // any valid reason (both)</p> <p>$0 \leq x \leq 5$</p> <p>$-4 \leq f(x) \leq 6$</p> <p>$f : x \rightarrow 2x - 4$ or $f(x) = 2x - 4$</p>	<p><input type="checkbox"/> N1 No Multiple Answer (NMA)</p> <p><input type="checkbox"/> N1 NMA</p> <p><input type="checkbox"/> N1 NMA</p> <p><input type="checkbox"/> N1 NMA</p>	<p>1</p> <p>3</p>	<p>4</p>
<p>4</p> <p>(a)</p> <p>(b)</p> <p>(i)</p> <p>(ii)</p>	<p>$\frac{5+2f(x)}{3} = \frac{2}{x} - 1$ OR $g^{-1}gf(x) = \frac{3\left(\frac{2}{x}-1\right)-5}{2}$</p> <p>$\left(\frac{3-4(3)}{3}\right)$</p> <p>-3</p> <p>$h(x) = \frac{x+7}{4}$</p> <p>$k+5 = \frac{p+7}{4}$</p> <p>$p = 4k + 13$</p>	<p><input type="radio"/> K1 Use valid method to find $f(x)$</p> <p><input type="radio"/> K1 Substitute $x = 3$ into $*f(x)$.</p> <p><input type="checkbox"/> N1 -3</p> <p><input type="checkbox"/> N1 $h(x) = \frac{x+7}{4}$</p> <p><input type="radio"/> K1 $k+5 = \frac{p+7}{4}$</p> <p><input type="checkbox"/> N1 $p = 4k + 13$</p>	<p>3</p> <p>3</p>	<p>6</p>
<p>5</p> <p>(a)</p> <p>(b)</p>	<p>$0 < a < 1$</p> <p>$7^{2x} \times 7^{2y}$ or $\frac{7^x}{7^y}$</p> <p>$7^{2x} \times 7^{2y} + \frac{7^x}{7^y}$ or equivalent</p> <p>$a^2b^2 + \frac{a}{b}$ or equivalent</p>	<p><input type="checkbox"/> P1 NMA</p> <p><input type="radio"/> K1 Use any one rule correctly</p> <p>$a^m \times a^n = a^{m+n}$</p> <p>$a^m \div a^n = a^{m-n}$</p> <p>$(a^m)^n = a^{mn}$</p> <p><input type="radio"/> K1 Use any two rules</p> <p><input type="checkbox"/> N1</p>	<p>1</p> <p>3</p>	<p>4</p>

<p>6</p> <p>(a) 1477.81</p> <p>(b) $200e^n + 20e^{n+1} > 800000$ $200e^n + 20[e^n(e^1)] > 800000$ $e^n(200 + 20e) > 800000$ $e^n > 3145.08$ $n > \ln(3145.08)$ $n > 8.05$ $n = 9$</p>		<p>N1</p> <p>K1 Use $a^m \times a^n = a^{m+n}$</p> <p>K1</p> <p>Use $e^a = b \Leftrightarrow a = \ln b$</p> <p>N1 9</p>	<p>1</p> <p>3</p>	<p>4</p>
<p>7</p> <p>(a) Discrete, countable (both)</p> <p>(b) (i)</p> <p>${}^{10}C_4 \left(\frac{2}{7}\right)^4 \left(\frac{5}{7}\right)^6$</p> <p>0.1859</p> <p>(ii)</p> <p>${}^{10}C_0 \left(\frac{5}{7}\right)^0 \left(\frac{2}{7}\right)^{10}$ or ${}^{10}C_1 \left(\frac{5}{7}\right)^1 \left(\frac{2}{7}\right)^9$ or ${}^{10}C_2 \left(\frac{5}{7}\right)^2 \left(\frac{2}{7}\right)^8$</p> <p>${}^{10}C_8 \left(\frac{2}{7}\right)^8 \left(\frac{5}{7}\right)^2$ or ${}^{10}C_9 \left(\frac{2}{7}\right)^9 \left(\frac{5}{7}\right)^1$ or ${}^{10}C_{10} \left(\frac{2}{7}\right)^{10} \left(\frac{5}{7}\right)^0$</p> <p>${}^{10}C_0 \left(\frac{5}{7}\right)^0 \left(\frac{2}{7}\right)^{10} + {}^{10}C_1 \left(\frac{5}{7}\right)^1 \left(\frac{2}{7}\right)^9 + {}^{10}C_2 \left(\frac{5}{7}\right)^2 \left(\frac{2}{7}\right)^8$ or</p> <p>${}^{10}C_8 \left(\frac{2}{7}\right)^8 \left(\frac{5}{7}\right)^2 + {}^{10}C_9 \left(\frac{2}{7}\right)^9 \left(\frac{5}{7}\right)^1 + {}^{10}C_{10} \left(\frac{2}{7}\right)^{10} \left(\frac{5}{7}\right)^0$</p> <p>0.001114</p>		<p>N1 NMA</p> <p>K1</p> <p>N1 0.1859</p> <p>K1</p> <p>K1</p> <p>N1 0.001114</p>	<p>1</p> <p>5</p>	<p>6</p>

<p>8</p> <p>(a) $({}^{10}C_8 \times {}^8C_7)$ or $({}^{10}C_9 \times {}^8C_6)$ or $({}^{10}C_{10} \times {}^8C_5)$</p> <p>$({}^{10}C_8 \times {}^8C_7) + ({}^{10}C_9 \times {}^8C_6) + ({}^{10}C_{10} \times {}^8C_5)$</p> <p>696</p> <p>(b) $\frac{(7-1)!}{2}$ or 6P_4</p> <p>$\frac{(7-1)!}{2}$ and 6P_4, Yes (both)</p>		 	<p>3</p> <p>2</p>	<p>5</p>
<p>9</p> <p>(a) (i) $\sqrt{k^2 + 2^2} = \sqrt{29}$</p> <p>$\overline{PQ} = \overline{PO} + \overline{OQ}$</p> <p>$*5\mathbf{i} + 2\mathbf{j} = (3\mathbf{i} - 8\mathbf{j}) + (n\mathbf{i} + 10\mathbf{j})$ or $\begin{pmatrix} *5 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \end{pmatrix} + \begin{pmatrix} n \\ 10 \end{pmatrix}$</p> <p>$n = 2$</p> <p>(ii) $\begin{pmatrix} *2 \\ 10 \end{pmatrix}$</p> <p>(b) $\overline{KN} = k\overline{LM}$</p> <p>$\overline{KL} + \overline{LM} + \overline{MN} = k\overline{LM}$</p> <p>$3\mathbf{a} + (\mathbf{a} + \mathbf{b}) - (5\lambda\mathbf{a} + \lambda\mathbf{b}) = k(\mathbf{a} + \mathbf{b})$</p> <p>$(4 - 5\lambda)\mathbf{a} + (1 - \lambda)\mathbf{b} = k\mathbf{a} + k\mathbf{b}$</p> <p>$4 - 5\lambda = k$ or $1 - \lambda = k$</p> <p>$k = \frac{1}{4}$</p> <p>$\overline{KN} = \frac{1}{4}\overline{LM}$</p> <p>@</p> <p>$\overline{LM} = k\overline{KN}$</p> <p>$\overline{LM} = k(\overline{KL} + \overline{LM} + \overline{MN})$</p> <p>$\mathbf{a} + \mathbf{b} = k[3\mathbf{a} + (\mathbf{a} + \mathbf{b}) - (5\lambda\mathbf{a} + \lambda\mathbf{b})]$</p> <p>$\mathbf{a} + \mathbf{b} = (4 - 5\lambda)k\mathbf{a} + (1 - \lambda)k\mathbf{b}$</p> <p>$(4 - 5\lambda)k = 1$ or $(1 - \lambda)k = 1$</p> <p>$k = 4$</p> <p>$\overline{LM} = 4\overline{KN}$</p>		 	<p>4</p> <p>4</p>	<p>8</p>

10 (a)	$-\frac{4}{5} \text{ or } -\frac{1}{\sqrt{5}} \text{ or } -\frac{2}{\sqrt{5}}$ $\left(-\frac{4}{5}\right)\left(-\frac{1}{\sqrt{5}}\right) + \left(-\frac{3}{5}\right)\left(-\frac{2}{\sqrt{5}}\right)$ <p>2</p>	<p>P1 Seen anywhere</p> <p>K1 Use $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$</p> <p>N1</p>	3	
(b)	$\tan 60^\circ = \sqrt{3}$ $\frac{k - \sqrt{3}}{1 + k\sqrt{3}} = -1$ $k - \sqrt{3} = -1 - k\sqrt{3}$ $k(1 + \sqrt{3}) = \sqrt{3} - 1$ $k = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$ $k = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$ $k = 2 - \sqrt{3}$	<p>P1 $\tan 60^\circ = \sqrt{3}$</p> <p>K1 Use $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$</p> <p>K1 Multiply conjugate</p> <p>N1 $2 - \sqrt{3}$</p>	4	7

11	$2ax - \frac{b}{x^2}$ <p> $m_t(-1) = -1$ or $m_n(2a - b) = -1$ </p> <p> $* \left(2a(1) - \frac{b}{(1)^2} \right) = 1 \quad \text{and} \quad 5 = a(1)^2 + \frac{b}{(1)}$ </p> <p> $a = 2, b = 3$ </p> <p>Note:</p> <p> $\frac{d}{dx}(ax^n)$ the power of x reduced by 1, must correct for both terms. </p>	<p> K1 Differentiate $y = ax^2 + \frac{b}{x}$ wrt x </p> <p> K1 Use $m_1 m_2 = -1$ and find m_2 </p> <p> K1 Solve simultaneous equations </p> <p> N1 Both correct $a = 2, b = 3$ </p>	4	4
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12				
(a)	$f = \frac{15g}{g-15} // g = \frac{15f}{f-15}$ $\frac{df}{dg} = \frac{(g-15)(15) - 15g(1)}{(g-15)^2} \text{ or equivalent}$ $\frac{-225}{(20-15)^2} \times 1.2$ -10.8	<div style="border: 1px solid black; width: 40px; height: 20px; margin-bottom: 10px; display: flex; align-items: center; justify-content: center;">P1</div> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 5px;">K1</div> <div>Use quotient rule</div> </div> <div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 5px;">K1</div> <div>Use chain rule</div> </div> <div style="border: 1px solid black; width: 40px; height: 20px; display: flex; align-items: center; justify-content: center;">N1</div>	4	
(b)	<p>(i)</p> $\left[\frac{dy}{dx} \right] = -2x$ <p>(ii)</p> $\delta x = k$ $\frac{\delta y}{k} \approx -2(p)$ $\delta y = -2pk$ $q = (1 - p^2) + (-2pk)$ $q = 1 - p^2 - 2pk$	<div style="border: 1px solid black; width: 40px; height: 20px; margin-bottom: 10px; display: flex; align-items: center; justify-content: center;">N1</div> <div>NMA</div> <div style="border: 1px solid black; width: 40px; height: 20px; margin-bottom: 10px; display: flex; align-items: center; justify-content: center;">P1</div> <div>$\delta x = k$</div> <div style="border: 1px solid black; border-radius: 50%; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-bottom: 10px;">K1</div> <div>Use $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$</div> <div style="border: 1px solid black; width: 40px; height: 20px; display: flex; align-items: center; justify-content: center;">N1</div> <div>$q = 1 - p^2 - 2pk$</div>	4	
				8

<p>13</p> <p>(a) $760 - 710 = 710 - 660$ or equivalent</p> <p>Arithmetic</p>		<p>(K1) Use $T_n - T_{n-1}$</p> <p>(N1)</p>	<p>2</p>	
<p>(b) $660 + (n - 1)(50) > 1350$</p> <p>$n = 15$</p> <p>Jan 2049</p>		<p>(K1) Use $T_n = a + (n - 1)d > 1350$ to find n.</p> <p>(N1) 15</p> <p>(N1) Jan 2049</p>	<p>3</p>	
<p>(c)</p> <p>$s_{14} = \frac{14}{2} [2(20640) + (14 - 1)(1200)]$ or</p> <p>$s_{18} = \frac{18}{2} [2(660) + (18 - 1)(50)]$ or</p> <p>$s_4 = \frac{4}{2} [2(660) + (4 - 1)(50)]$ or</p> <p>$s_3 = \frac{3}{2} [2(660) + (3 - 1)(50)]$</p> <p>$12(s_4 - s_3) + 24(s_{18} - s_4)$ or</p> <p>$12(810) + s_{14}$</p> <p>407880</p>		<p>(K1) Use $S_n = \frac{n}{2} [2a + (n - 1)d]$ with correct a and d</p> <p>(K1) Any valid method to find the total rental.</p> <p>(N1) 407880</p>	<p>3</p>	<p>8</p>

<p>14</p> <p>(a)</p> <p>(i)</p> <p>(ii)</p> <p>(b)</p>	<p>$y^2 = y^3$</p> <p>(1,1)</p> <p>$\int_0^1 y^3 dy$ or $\int_0^1 y^2 dy$ AND substitute the limit OR</p> <p>$\int_0^1 x^{\frac{1}{3}} dx$ or $\int_0^1 x^{\frac{1}{2}} dx$ AND substitute the limit</p> <p>$\left[\frac{y^3}{3} \right]_0^1 - \left[\frac{y^4}{4} \right]_0^1$ OR $\left[\frac{x^{\frac{4}{3}}}{\left(\frac{4}{3}\right)} \right]_0^1 - \left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \right]_0^1$</p> <p>$\frac{1}{12}$</p> <p>$V_1 = \frac{1}{3}\pi(1)^2(1)$ or $\pi \int_0^1 (2-y)^2 dy$</p> <p>or $V_2 = \int_0^1 (y^2)^2 dy$</p> <p>$\frac{1}{3}\pi + \frac{1}{5}\pi$</p> <p>$\frac{8}{15}\pi$</p>	<p>K1 Equate and solve $y^2(1-y) = 0$ or $y^2(y-1) = 0$</p> <p>N1</p> <p>K1</p> <p>K1</p> <p>N1 $\frac{1}{12}$</p> <p>K1 Find volume V_1 or V_2</p> <p>K1 $V_1 + V_2$</p> <p>N1 $\frac{8}{15}\pi$</p>	<p>5</p> <p>3</p>	<p>8</p>
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15				
(a)	$(-3, 0.5)$	<div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">P1</div>	1	
(b)	$(-4, -2)$ or $(0, -2)$ $\frac{0.5 - (-2)}{-3 - (-4)}, \frac{0.5 - (-2)}{-3 - 0}$ (both) $1 = 2.5x + 8$ AND $1 = -\frac{5}{6}x - 2$ OR $\frac{1 - 0.5}{x - (-3)} = 2.5$ or equivalent AND $\frac{1 - 0.5}{x - (-3)} = -\frac{5}{6}$ or equivalent $VU = 0.4, UT = 0.8$ (both) 1:2	<div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 0 auto;">P1</div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> <div style="margin-right: 5px;">Find gradient for both straight line</div> </div> <div style="text-align: center; margin: 5px 0;"> </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> <div style="margin-right: 5px;">Substitute value of y to find x</div> </div> <div style="text-align: center; margin: 5px 0;"> </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-right: 5px;">N1</div> </div> <div style="text-align: center; margin: 5px 0;"> </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">N1</div> </div>	5	
(c)	$m_{QU} \times \frac{0 - (-2)}{-1 - (-4)} = -1$ $m_{QU} = -\frac{5}{6} \neq -\frac{3}{2}$ or $-\frac{5}{6} \times -\frac{3}{2} \neq -1$ therefore not perpendicular	<div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin-right: 5px;">K1</div> <div style="margin-right: 5px;">Find gradient and use $m_1 \times m_2 = -1$</div> </div> <div style="text-align: center; margin: 5px 0;"> </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-right: 5px;">N1</div> </div>	2	
				8